Friday 6 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Three points have coordinates $A(-3, 12, -7)$, $B(-2, 6, 9)$, $C(6, 0, -10)$. The plane P passes through the points A, B and C.

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$. Hence or otherwise find an equation for the plane P in the form $ax + by + cz = d$. **[5]**

The plane Q has equation $6x + 3y + 2z = 32$. The perpendicular from A to the plane Q meets Q at the point D. The planes P and Q intersect in the line L.

(ii) Find the distance AD. **[3]**

(iii) Find an equation for the line L. **[5]**

(iv) Find the shortest distance from A to the line L. **[6]**

(v) Find the volume of the tetrahedron ABCD. **[5]**

Option 2: Multi-variable calculus

2 A surface S has equation $g(x, y, z) = 0$, where $g(x, y, z) = x^2 + 3y^2 + 2z^2 + 2yz + 6xz - 4xy - 24$. $P(2, 6, -2)$ is a point on the surface S.

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. **[3]**

(ii) Find the equation of the normal line to the surface S at the point P. **[3]**

(iii) The point Q is on this normal line and close to P. At Q, $g(x, y, z) = h$, where h is small. Find, in terms of h, the approximate perpendicular distance from Q to the surface S. **[4]**

(iv) Find the coordinates of the two points on the surface at which the normal line is parallel to the y-axis. **[6]**

(v) Given that $10x - y + 2z = 6$ is the equation of a tangent plane to the surface S, find the coordinates of the point of contact. **[8]**

Option 3: Differential geometry

3 **(a)** A curve has intrinsic equation $s = 2\ln\left(\dfrac{\pi}{\pi - 3\psi}\right)$ for $0 \leqslant \psi < \frac{1}{3}\pi$, where s is the arc length measured from a fixed point P and $\tan\psi = \dfrac{dy}{dx}$. P is in the third quadrant. The curve passes through the origin O, at which point $\psi = \frac{1}{6}\pi$. Q is the point on the curve at which $\psi = \frac{3}{10}\pi$.

(i) Express ψ in terms of s, and sketch the curve, indicating the points O, P and Q. **[4]**

(ii) Find the arc length OQ. **[3]**

(iii) Find the radius of curvature at the point O. **[3]**

(iv) Find the coordinates of the centre of curvature corresponding to the point O. **[3]**

(b) **(i)** Find the surface area of revolution formed when the curve $y = \frac{1}{3}\sqrt{x}(x-3)$ for $1 \leqslant x \leqslant 4$ is rotated through 2π radians about the y-axis. **[7]**

(ii) The curve in part **(b)(i)** is one member of the family $y = \frac{1}{9}\lambda\sqrt{x}(x-\lambda)$, where λ is a positive parameter. Find the equation of the envelope of this family of curves. **[4]**

Option 4: Groups

4 The twelve distinct elements of an abelian multiplicative group G are

$$e, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b$$

where e is the identity element, $a^6 = e$ and $b^2 = e$.

(i) Show that the element a^2b has order 6. **[3]**

(ii) Show that $\{e, a^3, b, a^3b\}$ is a subgroup of G. **[3]**

(iii) List all the cyclic subgroups of G. **[6]**

You are given that the set

$$H = \{1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89\}$$

with binary operation multiplication modulo 90 is a group.

(iv) Determine the order of each of the elements 11, 17 and 19. **[4]**

(v) Give a cyclic subgroup of H with order 4. **[2]**

(vi) By identifying possible values for the elements a and b above, or otherwise, give one example of each of the following:

(A) a non-cyclic subgroup of H with order 12, **[3]**

(B) a non-cyclic subgroup of H with order 4. **[3]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

The speeds of vehicles are measured on a busy stretch of road and are categorised as A (not more than 30 mph), B (more than 30 mph but not more than 40 mph) or C (more than 40 mph).

- Following a vehicle in category A, the probabilities that the next vehicle is in categories A, B, C are 0.9, 0.07, 0.03 respectively.
- Following a vehicle in category B, the probabilities that the next vehicle is in categories A, B, C are 0.3, 0.6, 0.1 respectively.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

This is modelled as a Markov chain with three states corresponding to the categories A, B, C. The speed of the first vehicle is measured as 28 mph.

(i) Write down the transition matrix **P**. **[2]**

(ii) Find the probabilities that the 10th vehicle is in each of the three categories. **[3]**

(iii) Find the probability that the 12th and 13th vehicles are in the same category. **[4]**

(iv) Find the smallest value of n for which the probability that the nth and $(n+1)$th vehicles are in the same category is less than 0.8, and give the value of this probability. **[4]**

(v) Find the expected number of vehicles (including the first vehicle) in category A before a vehicle in a different category. **[2]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity, and hence write down the equilibrium probabilities for the three categories. **[3]**

(vii) Find the probability that, after many vehicles have passed by, the next three vehicles are all in category A. **[2]**

On a new stretch of road, the same categories are used but some of the transition probabilities are different.

- Following a vehicle in category A, the probability that the next vehicle is in category B is equal to the probability that it is in category C.
- Following a vehicle in category B, the probability that the next vehicle is in category A is equal to the probability that it is in category C.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

In the long run, the proportions of vehicles in categories A, B, C are 50%, 40%, 10% respectively.

(viii) Find the transition matrix for the new stretch of road. **[4]**

END OF QUESTION PAPER

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1 (i)	
1 (ii)	

1 (iii)	

1 (iv)	

1 (v)	

2 (i)	
2 (ii)	
2 (iii)	

2 (iv)	

2 (v)	

3 (a) (i)	
3 (a) (ii)	

3 (a) (iii)	
3 (a) (iv)	

3 (b) (i)	

3 (b) (ii)	

4 (i)	
4 (ii)	

4 (iii)	

4 (iv)	
4 (v)	

4 (vi) (*A*)	
4 (vi) (*B*)	

5 (i)	
5 (ii)	
5 (iii)	

5 (iv)	
5 (v)	

5 (vi)	
5 (vii)	

5 (viii)	

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

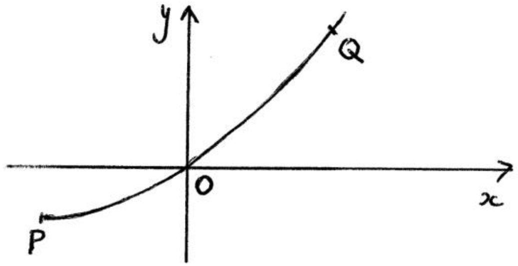
Question			Answer	Marks	Guidance	
1	(i)		$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}1\\-6\\16\end{pmatrix}\times\begin{pmatrix}9\\-12\\-3\end{pmatrix}=\begin{pmatrix}210\\147\\42\end{pmatrix}$ $\left[=21\begin{pmatrix}10\\7\\2\end{pmatrix}\right]$	M1 A2	Evaluation of vector product Give A1 for one correct element	One correct element (FT) Give A1 for a non-zero multiple
			Equation of P is $10x+7y+2z=d$ $10x+7y+2z=40$	M1 A1 **[5]**	*Accept* $210x+147y+42z-840=0$ *etc*	
1	(ii)		$\text{AD}=\left\lvert\dfrac{6(-3)+3(12)+2(-7)-32}{\sqrt{6^2+3^2+2^2}}\right\rvert$	M1 M1	For numerator For denominator	*M0 if constant term omitted*
			$=\dfrac{28}{7}=4$	A1 **[3]**		
		OR	$6(-3+6\lambda)+3(12+3\lambda)+2(-7+2\lambda)=32$		M1 Equation for λ	
			$\lambda=\dfrac{4}{7},\quad \text{AD}=\left\lvert\lambda\begin{pmatrix}6\\3\\2\end{pmatrix}\right\rvert=\dfrac{4}{7}\sqrt{6^2+3^2+2^2}$		M1 Using λ to find the distance AD *Independent of previous M1*	*But M0 if* $\lambda=\pm1$ *or* $\lambda=0$
			$=4$		A1	

Question			Answer	Marks	Guidance	
1	(iii)		When $x=0$, $7y+2z=40$, $3y+2z=32$ $\quad y=2,\ z=13$	M1 A1	Finding a point on L One correct point e.g. $(1,\ 1,\ 11.5)$	e.g. $(2,\ 0,\ 10)$, $(\frac{26}{3},\ -\frac{20}{3},\ 0)$
			$\begin{pmatrix}10\\7\\2\end{pmatrix}\times\begin{pmatrix}6\\3\\2\end{pmatrix}=\begin{pmatrix}8\\-8\\-12\end{pmatrix}$	M1 A1	Vector product of direction vectors Direction of L correct	OR Finding a second point on L and using 2 points to find direction
			Equation of L is $\mathbf{r}=\begin{pmatrix}0\\2\\13\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-3\end{pmatrix}$	A1 FT **[5]**	Any correct form *Dependent on M1M1*	Condone omission of ‘ $\mathbf{r}=$ ‘
		OR	Eliminating z, $4x+4y=8$ $x=\lambda,\ y=2-\lambda,\ z=13-\frac{3}{2}\lambda$		M1 Eliminating one variable A1 M1 Finding (e.g.) y and z in terms of x A1A1	Or $6y-4z=-40$ or $12x+8z=104$ Or A1 FT *dependent on M1M1*
1	(iv)		$\left[\begin{pmatrix}-3\\12\\-7\end{pmatrix}-\begin{pmatrix}0\\2\\13\end{pmatrix}\right]\times\begin{pmatrix}2\\-2\\-3\end{pmatrix}=\begin{pmatrix}-3\\10\\-20\end{pmatrix}\times\begin{pmatrix}2\\-2\\-3\end{pmatrix}=\begin{pmatrix}-70\\-49\\-14\end{pmatrix}$	M1 A2 FT	Appropriate vector product Give A1 if one error	
			Shortest distance is $\dfrac{\sqrt{70^2+49^2+14^2}}{\sqrt{2^2+2^2+3^2}}=\sqrt{\dfrac{7497}{17}}$	M1 M1	Finding magnitude of vector product Complete method for finding distance	*Dependent on previous M1* *Dependent on previous M1M1*
			Shortest distance is 21	A1 **[6]**	*A0 for 21 resulting from wrong v.p.*	
		OR	$\left[\begin{pmatrix}2\lambda\\2-2\lambda\\13-3\lambda\end{pmatrix}-\begin{pmatrix}-3\\12\\-7\end{pmatrix}\right].\begin{pmatrix}2\\-2\\-3\end{pmatrix}=0$		M1 Allow one error A1 FT	
			$\lambda=2$		M1 Obtaining a value of λ A1 FT	*Dependent on previous M1*
			Shortest distance is $\sqrt{(7)^2+(-14)^2+(14)^2}$ Shortest distance is 21		M1 A1	*Dependent on previous M1M1*

Question			Answer	Marks	Guidance	
1	(v)		$\overrightarrow{AD} = (\pm)\ \frac{4}{7}\begin{pmatrix}6\\3\\2\end{pmatrix}$	M1 A1 FT	$\overrightarrow{AD}$ is a multiple of $\begin{pmatrix}6\\3\\2\end{pmatrix}$ FT from (ii)	*M1 for* $\overrightarrow{AD} = \begin{pmatrix}6\\3\\2\end{pmatrix}$
			Volume is $\frac{1}{6}(\overrightarrow{AB} \times \overrightarrow{AC}) . \overrightarrow{AD}$	M1	Appropriate scalar triple product	*Just stated.* $\frac{1}{6}$ *not needed*
			$= \frac{1}{6} \times 21\begin{pmatrix}10\\7\\2\end{pmatrix} . \frac{4}{7}\begin{pmatrix}6\\3\\2\end{pmatrix} = 2(60+21+4)$	M1	Evaluation of scalar triple product	*Independent of previous M's, but must be numerical*
			$= 170$	A1 **[5]**		

Question			Answer	Marks	Guidance	
2	(i)		$\frac{\partial g}{\partial x} = 2x + 6z - 4y$	B1		
			$\frac{\partial g}{\partial y} = 6y + 2z - 4x$	B1		
			$\frac{\partial g}{\partial z} = 4z + 2y + 6x$	B1		
				[3]		
2	(ii)		At P, $\frac{\partial g}{\partial x} = -32,\ \frac{\partial g}{\partial y} = 24,\ \frac{\partial g}{\partial z} = 16$	B1		
			Normal line is $\mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 3 \\ 2 \end{pmatrix}$	M1	Direction of normal line	
				A1	FT	Condone omission of ' **r** = '
				[3]		
2	(iii)		$(h = \delta g \approx) \frac{\partial g}{\partial x}\delta x + \frac{\partial g}{\partial y}\delta y + \frac{\partial g}{\partial z}\delta z$	M1		
			$h = (-32)(-4\lambda) + (24)(3\lambda) + (16)(2\lambda) \quad (= 232\lambda)$	A1 FT		
			Approx distance is $\lvert \lambda \rvert \sqrt{4^2 + 3^2 + 2^2}$	M1		
			$= \sqrt{29}\lvert \lambda \rvert = \frac{\sqrt{29}\lvert h \rvert}{232}$	A1	Accept $\frac{h}{8\sqrt{29}},\ \frac{h}{43.1},\ 0.023h$ etc	
				[4]		

Question			Answer	Marks	Guidance	
2	(iv)		Require $\frac{\partial g}{\partial x} = \frac{\partial g}{\partial z} = 0$	M1		
			$2x + 6z - 4y = 0$ and $4z + 2y + 6x = 0$			
			$y = -x, \quad z = -x$	M1	For (e.g.) y and z as multiples of x	
			$x^2 + 3x^2 + 2x^2 + 2x^2 - 6x^2 + 4x^2 - 24 = 0$	M1	Quadratic in one variable	
			$6x^2 - 24 = 0$	A1	In simplified form	
			Points $(2, -2, -2)$ and $(-2, 2, 2)$	A1A1	If neither point correct, give A1 for any four correct coordinates	
				[6]		
2	(v)		$\begin{pmatrix} 2x + 6z - 4y \\ 6y + 2z - 4x \\ 4z + 2y + 6x \end{pmatrix} = \lambda \begin{pmatrix} 10 \\ -1 \\ 2 \end{pmatrix}$	M1	Allow M1 even if $\lambda = 1$	
				A1 FT		
			$y = 3x, \quad z = -5x$	M1	For (e.g.) y and z as multiples of x	Or $x = -\frac{1}{4}\lambda, \; y = -\frac{3}{4}\lambda, \; z = \frac{5}{4}\lambda$
			$x^2 + 27x^2 + 50x^2 - 30x^2 - 30x^2 - 12x^2 - 24 = 0$	M1	Quadratic in one variable	
			$6x^2 - 24 = 0$	A1	Or $y^2 - 36 = 0$ or $z^2 - 100 = 0$	Or $\lambda^2 - 64 = 0$
			Possible points $(2, 6, -10)$ and $(-2, -6, 10)$	A1	For one correct point	
			At $(2, 6, -10)$, $10x - y + 2z = -6$ At $(-2, -6, 10)$, $10x - y + 2z = 6$	M1	Checking at least one point	
			It is the tangent plane at $(-2, -6, 10)$	A1		
				[8]		
		OR	$10x - (3x) + 2(-5x) = 6$ $x = -2$		M1 Equation in one variable A1 Or $y = -6$ or $z = 10$ or $\lambda = 8$ M1 Using this value to obtain at least two coordinates	
			It is the tangent plane at $(-2, -6, 10)$		A2 Give A1 for two coordinates correct	

Question			Answer	Marks	Guidance	
3	(a)	(i)	$\psi = \frac{1}{3}\pi(1-e^{-\frac{s}{2}})$ 	B1 B1 B1 B1 **[4]**	 Positive increasing gradient through O Zero gradient at P Q marked in first quadrant	
3	(a)	(ii)	At O ($\psi = \frac{1}{6}\pi$), $s = 2\ln 2$ At Q ($\psi = \frac{3}{10}\pi$), $s = 2\ln 10$ Arc length OQ is $2\ln 10 - 2\ln 2 = 2\ln 5$	M1 M1 A1 **[3]**	 *Or* $\ln 25$ *or* 3.22 *(only)*	
3	(a)	(iii)	$\rho = \frac{ds}{d\psi}$ $= \frac{6}{\pi - 3\psi}$ At O ($\psi = \frac{1}{6}\pi$), radius of curvature is $\rho = \frac{12}{\pi}$	M1 A1 A1 **[3]**	*Or* $(\kappa =) \frac{d\psi}{ds}$ *Or* $\kappa = \frac{\pi}{6}e^{-\frac{s}{2}}$ *and* $s = 2\ln 2$ Accept 3.82	 *All 3 marks can be awarded in (iv)*
3	(a)	(iv)	Centre of curvature is $(-\rho\sin\psi,\ \rho\cos\psi)$ $\left(-\frac{6}{\pi},\ \frac{6\sqrt{3}}{\pi}\right)$	M1 A1A1 **[3]**	 FT is $-\frac{1}{2}\lvert\rho\rvert,\ \frac{\sqrt{3}}{2}\lvert\rho\rvert$	 Accept $(-1.91,\ 3.31)$

Question			Answer	Marks	Guidance	
3	(b)	(i)	$1+\left(\frac{dy}{dx}\right)^2 = 1+\left(\frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}\right)^2$	B1		
			$= 1+\frac{1}{4}x - \frac{1}{2} + \frac{1}{4}x^{-1} = \frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1}$	M1		
			$= \left(\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)^2$	A1	*or* $\frac{(x+1)^2}{4x}$	*Condone correct answer from inaccurate working*
			Area is $\int 2\pi x \, ds$	M1		
			$= \int_1^4 2\pi x\left(\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)dx$	A1	Any correct form	
			$= \pi\left[\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\right]_1^4$	A1		
			$= \frac{256}{15}\pi$	A1	*Exact answer only*	
				[7]		
3	(b)	(ii)	Differentiating partially with respect to λ	M1		
			$0 = \frac{1}{9}x^{\frac{3}{2}} - \frac{2}{9}\lambda x^{\frac{1}{2}}$	A1	*For RHS*	
			$\lambda = \frac{1}{2}x$, so $y = \frac{1}{9}(\frac{1}{2}x)x^{\frac{3}{2}} - \frac{1}{9}(\frac{1}{4}x^2)x^{\frac{1}{2}}$	M1	Eliminating λ	
			$y = \frac{1}{36}x^{\frac{5}{2}}$	A1	*Must be simplified*	
				[4]		

Question			Answer	Marks	Guidance	
4	(i)		$(a^2b)^2 = a^4b^2 = a^4$	M1	Finding one power	
			$(a^2b)^3 = b$, $(a^2b)^4 = a^2$, $(a^2b)^5 = a^4b$	A1	Three powers correct	
			$(a^2b)^6 = e$			
			Hence a^2b has order 6	E1	Fully correct explanation	*No need to state conclusion, provided it has been fully justified*
				[3]		
4	(ii)		<table><tr><td></td><td>e</td><td>a^3</td><td>b</td><td>a^3b</td></tr><tr><td>e</td><td>e</td><td>a^3</td><td>b</td><td>a^3b</td></tr><tr><td>a^3</td><td>a^3</td><td>e</td><td>a^3b</td><td>b</td></tr><tr><td>b</td><td>b</td><td>a^3b</td><td>e</td><td>a^3</td></tr><tr><td>a^3b</td><td>a^3b</td><td>b</td><td>a^3</td><td>e</td></tr></table>	B2	Give B1 for no more than three errors or omissions	
			The set is closed; hence it is a subgroup of G	B1	*'Closed' (or equivalent) is required*	
				[3]		
4	(iii)		$\{e, a^3\}$, $\{e, b\}$, $\{e, a^3b\}$	B2	Give B1 for one correct	*Deduct one mark (out of B2) for each set of order 2 in excess of 3*
			$\{e, a^2, a^4\}$	B1	B0 if any other set of order 3	
			$\{e, a, a^2, a^3, a^4, a^5\}$	B1		
			$\{e, a^2b, a^4, b, a^2, a^4b\}$	B1		
			$\{e, ab, a^2, a^3b, a^4, a^5b\}$	B1		*Deduct one mark (out of B3) for each set of order 6 in excess of 3*
					No mark for $\{e\}$. Deduct one mark (out of B6) for each set (including G) of order other than 1, 2, 3, 6	
				[6]		

Question			Answer	Marks	Guidance	
4	(iv)		$11^2=31,\ 11^3=71,\ 11^4=61,\ 11^5=41,\ 11^6=1$ $17^2=19,\ 17^3=53,\ 17^4=1$ 11 has order 6 17 has order 4 $19^2=1$; 19 has order 2	M1 A1 A1 B1 **[4]**	Finding at least two powers of 11 (or 17) *Either correct implies M1*	
4	(v)		$\{1,\ 17,\ 19,\ 53\}$	M1 A1 **[2]**	Selecting powers of 17 Or B2 for $\{1,\ 37,\ 19,\ 73\}$	
4	(vi)	(*A*)	Taking $a=11,\ b=19$ $1, 11, 11^2, \ldots, 11^5, 19, 11\times 19, 11^2\times 19, \ldots, 11^5\times 19$ $\{1, 11, 31, 71, 61, 41, 19, 29, 49, 89, 79, 59\}$ i.e. $\{1, 11, 19, 29, 31, 41, 49, 59, 61, 71, 79, 89\}$	B1 M1 A1 **[3]**	*There are (many) other possibilities* Finding elements of *G* using their *a*, *b*	
4	(vi)	(*B*)	$1,\ 11^3,\ 19,\ 11^3\times 19$ $\{1,\ 71,\ 19,\ 89\}$	M1 M1 A1 **[3]**	Reference to group in (ii) Finding group in (ii) with their *a*, *b*	

Question			Answer	Marks	Guidance	
5			***Pre-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	*Do not penalise answers given to more than 4 dp*
5	(i)		$\mathbf{P} = \begin{pmatrix} 0.9 & 0.3 & 0.1 \\ 0.07 & 0.6 & 0.7 \\ 0.03 & 0.1 & 0.2 \end{pmatrix}$	B2 **[2]**	Give B1 for two columns correct	
5	(ii)		$\mathbf{P}^9 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ P(A) = 0.7268 P(B) = 0.2189 P(C) = 0.0544	M1 M1 A1 **[3]**	For $\mathbf{P}^9$ (allow $\mathbf{P}^{10}$) For initial column matrix (or first column of $\mathbf{P}^9$)	 *Dependent on previous M1*
5	(iii)		$\mathbf{P}^{11} = \begin{pmatrix} 0.7242 & ... & ... \\ 0.2211 & ... & ... \\ 0.0547 & ... & ... \end{pmatrix}$ $0.7242 \times 0.9 + 0.2211 \times 0.6 + 0.0547 \times 0.2$ $= 0.7954$	M1 M1 M1 A1 **[4]**	Appropriate elements from $\mathbf{P}^{11}$ Diagonal elements from **P** *Dependent on previous M1M1*	(allow $\mathbf{P}^{12}$)
5	(iv)		$\begin{pmatrix} 0.9 & 0.6 & 0.2 \end{pmatrix} \mathbf{P}^{n-1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $= (0.8009)$ when $n = 7$ $= (0.7986)$ when $n = 8$ Smallest value is $n = 8$ Probability is 0.7986	M1 M1 B1 A1 **[4]**	Repeating (iii) for another value of n Obtaining values both sides of 0.8 *0.7986 implies M1M1*	 Valid method required here

Question			Answer	Marks	Guidance	
5	(v)		Expected run length is $\frac{1}{1-0.9}$ $= 10$	M1 A1 **[2]**	Using $\frac{1}{1-p}$ or $\frac{p}{1-p}$ with $p=0.9$	
5	(vi)		$\mathbf{P}^n \to \begin{pmatrix} 0.7225 & 0.7225 & 0.7225 \\ 0.2225 & 0.2225 & 0.2225 \\ 0.0549 & 0.0549 & 0.0549 \end{pmatrix}$ P(A) = 0.7225 P(B) = 0.2225 P(C) = 0.0549	B2 B1 **[3]**	Give B1 for 6 elements correct to 3 dp FT if columns agree to 4 dp	
5	(vii)		$0.7225 \times 0.9 \times 0.9$ $= 0.5853$	M1 A1 **[2]**	FT P(A) × 0.81	
5	viii		$\begin{pmatrix} 1-2x & y & 0.1 \\ x & 1-2y & 0.7 \\ x & y & 0.2 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.4 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.4 \\ 0.1 \end{pmatrix}$ $0.5(1-2x)+0.4y+0.01=0.5$ $0.5x+0.4(1-2y)+0.07=0.4$ $0.5x+0.4y+0.02=0.1$ $x=0.06, \quad y=0.125$ Transition matrix is $\begin{pmatrix} 0.88 & 0.125 & 0.1 \\ 0.06 & 0.75 & 0.7 \\ 0.06 & 0.125 & 0.2 \end{pmatrix}$	M1 A1 M1 A1 **[4]**	First or second column correct Obtaining values for *x* and *y*	

Question			Answer	Marks	Guidance	
5			***Post-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	*Do not penalise answers given to more than 4 dp*
5	(i)		$\mathbf{P} = \begin{pmatrix} 0.9 & 0.07 & 0.03 \\ 0.3 & 0.6 & 0.1 \\ 0.1 & 0.7 & 0.2 \end{pmatrix}$	B2 **[2]**	Give B1 for two rows correct	
5	(ii)		$(1 \quad 0 \quad 0)\,\mathbf{P}^9$ P(A) = 0.7268 P(B) = 0.2189 P(C) = 0.0544	M1 M1 A1 **[3]**	For $\mathbf{P}^9$ (allow $\mathbf{P}^{10}$) For initial row matrix (or first row of $\mathbf{P}^9$)	*Dependent on previous M1*
5	(iii)		$\mathbf{P}^{11} = \begin{pmatrix} 0.7242 & 0.2211 & 0.0547 \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix}$ $0.7242 \times 0.9 + 0.2211 \times 0.6 + 0.0547 \times 0.2$ $= 0.7954$	M1 M1 M1 A1 **[4]**	Appropriate elements from $\mathbf{P}^{11}$ Diagonal elements from **P** *Dependent on previous M1M1*	(allow $\mathbf{P}^{12}$)
5	(iv)		$(1 \quad 0 \quad 0)\,\mathbf{P}^{n-1}\begin{pmatrix} 0.9 \\ 0.6 \\ 0.2 \end{pmatrix}$ $= (0.8009)$ when $n = 7$ $= (0.7986)$ when $n = 8$ Smallest value is $n = 8$ Probability is 0.7986	M1 M1 B1 A1 **[4]**	Repeating (iii) for another value of n Obtaining values both sides of 0.8 *0.7986 implies M1M1*	Valid method required here

Question			Answer	Marks	Guidance	
5	(v)		Expected run length is $\frac{1}{1-0.9}$ $= 10$	M1 A1 **[2]**	Using $\frac{1}{1-p}$ or $\frac{p}{1-p}$ with $p=0.9$	
5	(vi)		$\mathbf{P}^n \to \begin{pmatrix} 0.7225 & 0.2225 & 0.0549 \\ 0.7225 & 0.2225 & 0.0549 \\ 0.7225 & 0.2225 & 0.0549 \end{pmatrix}$ P(A) = 0.7225 P(B) = 0.2225 P(C) = 0.0549	B2 B1 **[3]**	Give B1 for 6 elements correct to 3 dp FT if rows agree to 4 dp	
5	(vii)		$0.7225 \times 0.9 \times 0.9$ $= 0.5853$	M1 A1 **[2]**	FT P(A) × 0.81	
5	viii		$0.5 \quad 0.4 \quad 0.1 \begin{pmatrix} 1-2x & x & x \\ y & 1-2y & y \\ 0.1 & 0.7 & 0.2 \end{pmatrix}$ $= 0.5 \quad 0.4 \quad 0.1$ $0.5(1-2x)+0.4y+0.01=0.5$ $0.5x+0.4(1-2y)+0.07=0.4$ $0.5x+0.4y+0.02=0.1$ $x=0.06, \quad y=0.125$ Transition matrix is $\begin{pmatrix} 0.88 & 0.06 & 0.06 \\ 0.125 & 0.75 & 0.125 \\ 0.1 & 0.7 & 0.2 \end{pmatrix}$	M1 A1 M1 A1 **[4]**	First or second row correct Obtaining values for x and y	

4757 Further Applications of Advanced Mathematics (FP3)

General Comments:

The candidates for this paper exhibited a wide range of ability; there were a few with very low marks, and several with full marks. Most candidates were well prepared and were able to demonstrate their knowledge and technical competence within their chosen topics. Q.1 and Q.2 were considerably more popular than Q.3, Q.4 and Q.5. Almost all candidates appeared to have sufficient time to make complete attempts at three questions, and only a few offered attempts at more than the required three questions.

Comments on Individual Questions:

Question No. 1 (Vectors)

Most candidates attempted this question and showed good knowledge of the relevant techniques.

In part (i) the vector product was usually calculated accurately and almost all candidates knew how to find the equation of the plane. Some candidates 'simplified' the vector product (for example by dividing it by 21) and this was penalised if the correct value did not appear anywhere.

In part (ii) the perpendicular distance from a point to a plane was usually found efficiently and correctly. The most common error was an incorrect sign for the constant term (32), and some candidates used the equation of the plane *P* instead of *Q*.

In part (iii) most candidates could find the line of intersection of two planes.

In part (iv) the perpendicular distance from a point to a line was usually found confidently as the magnitude of a vector product, with some candidates using alternative methods.

In part (v) most candidates wrote down a suitable scalar triple product and showed that they knew how to evaluate such a product. One of the vectors needed was AD and very many candidates had difficulty finding this, often confusing it with the position vector OD.

Question No. 2 (Multi-variable calculus)

This question was also attempted by most candidates, and all parts except (v) were generally well answered.

The partial differentiation in part (i) and finding the normal line in part (ii) were done correctly by almost all the candidates.

In part (iii) most candidates used $g_x\delta x + g_y\delta y + g_z\delta z$ to obtain an approximate linear relationship between h and the parameter in the normal line, but many did not go on to find the distance.

In part (iv) most candidates used $g_x = g_z = 0$ to obtain $x : y : z = 1 : -1 : -1$. It was then necessary to substitute into the equation of *S*; many candidates thought that $g_y = 1$ and used this instead.

In part (v) the first step is to show that $g_x : g_y : g_z = 10 : -1 : 2$ leads to $x : y : z = 1 : 3 : -5$. Then substituting into the equation of the tangent plane gives the required point (−2, −6, 10). If the equation of *S* is used two possible points are obtained and it is then necessary to check which of these lies on the given tangent plane. A very large number of candidates thought that $g_x = 10$, $g_y = -1$ and $g_z = 2$, giving the point (−1/4, −3/4, 5/4). This earned just 1 mark out of 8.

Question No. 3 (Differential geometry)

Candidates who attempted this question usually showed good understanding of the relevant techniques.

In part (a)(i) the equation was usually rearranged correctly and there were some good sketches of the curve. The sketch needed to show a zero gradient at P, the curve passing through O with a positive and increasing gradient, and Q marked in the first quadrant. Many candidates did not attempt the sketch at all.

In part (a)(ii) the arc length was very often found correctly. Some candidates just evaluated *s* at Q which gives the arc length PQ instead of the required OQ.

In part (a)(iii) most candidates indicated that they needed to find $ds/d\psi$ (or $d\psi/ds$) but many were unable to differentiate accurately. Those who first wrote *s* in the form $2\ln\pi - 2\ln(\pi - 3\psi)$ were more successful than the others.

In part (a)(iv) almost all candidates understood how to use their radius of curvature to find the required centre of curvature.
Part (b)(i) was well answered, with many candidates finding the curved surface area correctly. Some used the formula for rotation about the *x*-axis instead of the *y*-axis.

In part (b)(ii) the method for finding the envelope was very well understood.

Question No. 4 (Groups)

Most of the candidates who attempted this question were able to demonstrate a sound understanding of the topics tested.

Part (i) was generally well answered. As well as showing that $(a^2b)^6 = e$ it was necessary to show that no lower power was the identity, and not all candidates did this.

Part (ii) was very well done, usually by writing out the composition table for the subgroup.

In part (iii) there are three cyclic subgroups of order 2, one of order 3 and three of order 6. It was common for some of these to be omitted (notably those of order 6), and some candidates included the subgroup from part (ii) or *G* (which are non-cyclic) in their list.

Parts (iv) and (v) were almost always answered correctly.

In part (vi) most candidates identified $a = 11$ and $b = 19$ and often obtained the correct non-cyclic subgroups.

Question No. 5 (Markov chains)

The candidates who attempted this question usually answered it well. They were very competent at using their calculators, and powers of matrices were almost invariably evaluated correctly.

In part (i) the transition matrix **P** was almost always correct.

In part (ii) the probabilities were usually obtained correctly. Some candidates used $\mathbf{P}^{10}$ instead of $\mathbf{P}^{9}$.

In part (iii) most candidates used the probabilities for the 12th vehicle, but some combined these with the probabilities for the 13th vehicle instead of the diagonal elements from **P**.

There was a lot of good work in part (iv) with candidates repeating the work in part (iii) for different values of *n*. A common error was, after doing all the calculations correctly, to give the answer as $n = 7$ instead of $n = 8$.

In part (v) the expected number was often given as 9 instead of 10.

Part (vi) was well answered. Some candidates only gave the equilibrium probabilities and did not exhibit the limiting matrix as required by the question.

In part (vii) many candidates calculated 0.7225^3 or 0.7225×0.9^3 instead of 0.7225×0.9^2.

In part (viii) most candidates knew how to use the equilibrium probabilities to find the new transition matrix, and many completed this accurately.

Unit level raw mark and UMS grade boundaries June 2014 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0